

Simple Model for (3+2) Neutrino Oscillations

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Abstract

We formulate a set of naturalness criteria for sterile neutrinos (ν') to be light, needed for reconciling the LSND neutrino anomaly with the other neutrino data. A light sterile neutrino becomes as natural as the light active neutrinos if it carries quantum numbers of a chiral gauge symmetry broken at the TeV scale. The simplest such theory is shown to be an $SU(2)$ gauge theory with the ν' transforming as a spin 3/2 multiplet. We develop this model and show that it leads naturally to the phenomenologically viable (3+2) neutrino oscillation scheme. We also present next-to-minimal models for light sterile neutrinos based on a chiral $U(1)$ gauge symmetry.

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1 Introduction

Recent solar [1], atmospheric [2], and reactor [3] neutrino oscillation experiments have significantly improved our knowledge about neutrino masses and mixing angles. In particular, the solar and the atmospheric neutrino data are very well described in a three-neutrino oscillation scenario where the mass squared splittings are respectively $\Delta m_{\odot}^2 \simeq 7.5 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{\text{atm}}^2 \simeq 2.0 \times 10^{-3} \text{ eV}^2$ [4]. On the other hand, the $\bar{\nu}_{\mu}-\bar{\nu}_e$ oscillation signal reported by the Liquid Scintillator Neutrino Detector (LSND) experiment at Los Alamos [5], which will soon be tested by the ongoing MiniBooNE [6] experiment at Fermilab, would require a third neutrino mass squared splitting $\Delta m_{\text{LSND}}^2 \gtrsim 10^{-1} \text{ eV}^2$, which is impossible to implement in a three-neutrino oscillation scheme. Instead, one possibility to accomodate all the neutrino data is to add one or more light sterile neutrinos with masses of the order $\sim 1 \text{ eV}$, which would provide additional mass splittings. Although four-neutrino mass models with a single sterile neutrino [7, 8] are strongly disfavored by present data [9], a combined analysis of the short-baseline experiments Bugey [10], CCFR [11], CDHS [12], CHOOZ [13], KARMEN [14], and LSND shows, that (3+2) neutrino mass schemes with two sterile neutrinos can yield a satisfactory description of current neutrino oscillation data including LSND [15]. Generally, in (3+n) neutrino mass schemes, where n denotes the number of sterile neutrinos, it seems [15] that the LSND signal still remains compatible with the other data sets even when $n > 2$.

While the seesaw mechanism [16] provides a simple understanding of the smallness of active neutrino masses, it does not explain why a sterile neutrino ν' would be light. In fact, if the effective low-energy theory is the Standard Model (SM), then there is no reason why ν' would not acquire a mass of the order of the Planck scale $M_{Pl} \sim 10^{19} \text{ GeV}$. Thus, in any (3+n) neutrino mass model it is important to explain the smallness of the sterile neutrino masses. In this paper, we wish to formulate a set of naturalness criteria for light sterile neutrinos which would be as compelling as the seesaw mechanism for active neutrinos. We suggest and develop the simplest models which satisfy these criteria.

It is useful to recall the main ingredients that make the seesaw mechanism successful. Here, the set of left-handed SM neutrinos ν_e, ν_{μ} , and ν_{τ} is extended by introducing three right-handed neutrinos N_1, N_2 , and N_3 , which are singlets under the SM gauge group $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$. In the basis $(\nu_e, \nu_{\mu}, \nu_{\tau}, N_1, N_2, N_3)$, the resulting 6×6 neutrino mass matrix then reads

$$M_{\nu} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}, \quad (1)$$

where the entries 0, m_D , and M_R are 3×3 matrices which are characterized by the gauge-structure and the Higgs-content of the theory. It is significant that, in Eq. (1), the entries in the upper-left 3×3 sector are all vanishing. This is because the SM is a chiral gauge-theory and does not permit a bare mass term for the left-handed neutrinos. In addition, there are no Higgs triplet fields, which could have directly coupled to ν_i . Furthermore, the matrix elements of m_D are of the order of the electroweak scale $\sim 10^2 \text{ GeV}$ and protected from becoming too large by electroweak gauge invariance. In contrast to this, the mass matrix M_R has unprotected entries of the order M_{Pl} or of order the $B-L$ breaking scale $M_{B-L} \sim 10^{15} \text{ GeV}$. As a result, we obtain an effective 3×3 neutrino mass matrix $M_{\text{eff}} = -m_D M_R^{-1} m_D^T$, which leads to small neutrino masses of the order $\sim 10^{-2} \text{ eV}$.

By analogy with the seesaw mechanism for active neutrinos, we propose the following criteria for a light sterile neutrino ν' (with mass of order 1 eV) to be natural:

1. ν' must transform as a chiral representation of a “sterile” gauge symmetry G' which is broken at the TeV scale.
2. There must exist no Higgs field which couples directly to ν' .

Note that we require G' to be a gauge symmetry, rather than a global symmetry, since only gauge symmetries will survive quantum gravity corrections. In our constructions, we will supplement the above criteria by the requirement of a minimal Higgs sector: a single Higgs field breaks G' and provides simultaneously sterile neutrino masses, analogous to the SM Higgs doublet.

To illustrate the basic idea, let us consider the simplified case of one generation with one active neutrino flavor ν and one sterile neutrino ν' [(1+1) model]. Following our criteria, we extend the SM gauge symmetry to $G_{SM} \times G'$, with the ν' transforming chirally under G' . All SM particles carry zero G' charges. Next, we introduce two right-handed neutrinos N and N' , which are singlets under the total gauge group $G_{SM} \times G'$. In analogy with the electroweak symmetry breaking in the SM, we assume that G' is spontaneously broken around the TeV scale by a suitable Higgs field Φ which has no direct Yukawa coupling of the type $\nu'\nu'\Phi$. To keep the situation simple, we furthermore take Φ to be a singlet under G_{SM} . In the basis (ν, ν', N, N') , the total 4×4 neutrino mass matrix takes then the form

$$M_\nu = \begin{pmatrix} 0 & 0 & m_D & m_D'' \\ 0 & 0 & \tilde{m}_D'' & m_D' \\ m_D & \tilde{m}_D'' & M_R & M_R'' \\ m_D'' & m_D' & M_R' & M_R \end{pmatrix}. \quad (2)$$

We hence observe, that the general principles which lead to the usual seesaw mechanism, have also in this case dictated the canonical structure of M_ν in Eq. (1). Particularly, in Eq. (2), the vanishing of the mass terms in the upper-left 2×2 -block results from the chiral nature of the $G_{SM} \times G'$ gauge theory and the absence of specific Higgs representations. Moreover, m_D , m_D' , m_D'' , and \tilde{m}_D'' are of the order $\sim 10^2$ GeV, since they are protected by gauge invariance under G_{SM} and G' up to the TeV scale, where both G_{SM} and G' are spontaneously broken. The entries M_R , M_R' , and M_R'' on the other hand, are unprotected by $G_{SM} \times G'$ and thus of the order $\sim M_{B-L}$. At low energies, this will therefore give an effective 2×2 neutrino mass matrix, which yields small masses in the (sub-)eV-range for both the active and the sterile neutrinos. The generalization of this sterile neutrino seesaw mechanism to a $(3+n)$ mass scheme is straightforward with m_D becoming a $3 \times n$ matrix and M_R becoming an $n \times n$ matrix in Eq. (1). Notice that in the special case when G' is identified with a copy of G_{SM} , we arrive at the well-known scenario for “mirror” neutrinos [17]. Alternative ways of realizing light sterile neutrinos have been suggested in Ref. [18].

In this paper, we construct the simplest neutrino mass model consistent with our criteria for a light ν' . As it turns out, the simplest model yields the phenomenologically viable scenario of $(3+2)$ neutrino oscillations [15]. Here, we require invariance under the product group $G_{SM} \times G'$, where G' is a chiral anomaly-free continuous gauge symmetry. This implies,

in particular, that no extra discrete symmetry is imposed. The simplest example of this kind is found to be when $G' = SU(2)$, with the sterile neutrinos Ψ in the spin 3/2 representation. A single spin 3/2 Higgs field Φ can spontaneously break this symmetry at the TeV scale without supplying large (TeV scale) masses to Ψ . In this setup, we calculate the most general neutrino mass matrix M_ν by explicitly minimizing the scalar potential for Φ . The minimum of the potential preserves a Z_3 subgroup of the sterile isospin symmetry. The isospin $\pm\frac{3}{2}$ components of Ψ are neutral under this Z_3 , while the $\pm\frac{1}{2}$ components have charges ± 1 . Thus, only $\Psi_{\pm 3/2}$ will mix with the active neutrinos, yielding a (3+2) oscillation scheme. We also present the next simplest examples based on a chiral $U(1)$ gauge theory. Cancellation of chiral anomalies requires the existence of at least five – more naturally six – Weyl spinors, making these examples the second simplest.

2 A simple chiral $SU(2)$ model

The existence of a chiral gauge symmetry G' , broken at the TeV scale, plays a crucial rôle in our criteria for realizing naturally a light sterile neutrino. The vanishing of the axial vector anomalies and the mixed gauge-gravitational anomalies [19] sets non-trivial constraints on such a theory. We are naturally led to the choice $G' = SU(2)$, where these anomalies automatically vanish for any representation. Furthermore, $SU(2)$ admits chiral representations, *i.e.*, fermionic representations for which mass terms are forbidden by gauge invariance. Chiral $U(1)$ theories, while also interesting, are not the simplest as they require at least five spin 1/2 Weyl fermions for non-trivial anomaly cancellation. These next-to-minimal models are discussed in the next section.

Consider an $SU(2)$ gauge theory with one fermion field Ψ in the spin j representation. The spin j representation of $SU(2)$ yields for $j = 0, 1, 2, \dots$ (bosonic case) a unitary and for $j = 1/2, 3/2, 5/2, \dots$ (fermionic case) a projective unitary representation (with essential cocycle) of $SO(3)$. Although the axial vector anomalies and the mixed gauge-gravitational anomalies are zero for any j , the spin 1/2 representation of $SU(2)$ is plagued with a global Witten anomaly [20]. The spin 1 representation will not suit our needs as it is vectorial. The global $SU(2)$ anomaly vanishes, however, when Ψ transforms under the spin 3/2 representation, which has an even quadratic index. In this case, Ψ also cannot have an explicit mass term. Therefore, $SU(2)$ with a single fermion Ψ in the spin 3/2 representation is the simplest anomaly-free chiral gauge theory. $SU(2)$ with spin 3/2 matter fields has been studied in the context of dynamical supersymmetry breaking in Ref. [21]. Non-Abelian chiral gauge theories are necessary ingredients for dynamical supersymmetry breaking and have been analyzed extensively [22].

The gauge symmetry of our model is $G_{SM} \times SU(2)$. We will assume here that the $SU(2)$ symmetry is spontaneously broken by the vacuum expectation value (VEV) of a single Higgs field Φ at the TeV scale. Like the fermion Ψ , we put Φ into the spin 3/2 representation of the $SU(2)$ symmetry. In component form, one can write the $SU(2)$ spin 3/2 representations Ψ and Φ as $\Psi = (\psi_1, \psi_2, \psi_3, \psi_4)^T$ and $\Phi = (\phi_1, \phi_2, \phi_3, \phi_4)^T$, where ψ_1, ψ_2, ψ_3 , and ψ_4 denote 2-component Weyl spinors and ϕ_1, ϕ_2, ϕ_3 , and ϕ_4 are complex-valued¹ scalar fields. Here, we

¹Recall that the spin j representation of $SU(2)$ is defined on the space of polynomial functions on \mathbb{C}^2

take all SM particles to be singlets under the $SU(2)$ symmetry, while Ψ and Φ , on the other hand, are sterile with respect to G_{SM} . In addition, we assume seven right-handed neutrinos N_α ($\alpha = 1, \dots, 7$), which are total singlets under $G_{SM} \times SU(2)$. For n light ν' fields we will assume a total of $n + 3$ superheavy fields N_α . As a result of the product group-structure and the fermionic charge assignment, this model is automatically free of all anomalies.

The renormalizable Lagrangian relevant for neutrino masses is given by

$$\mathcal{L}_Y = a_{i\alpha} \ell_i H N_\alpha + b_\alpha \Psi \Phi^* N_\alpha + c_\alpha \Psi \Phi N_\alpha + M_{\alpha\beta} N_\alpha N_\beta + \text{h.c.}, \quad (3)$$

where H is the SM Higgs doublet, ℓ_i ($i = e, \mu, \tau$) denotes the SM lepton doublets, $a_{i\alpha}$, b_α , and c_α are Yukawa couplings of order unity and $M_{\alpha\beta}$ ($\alpha, \beta = 1, \dots, 7$) are of order $10^{14} - 10^{16} \text{ GeV}$. Note that Eq. (3) leads to a mass matrix structure as given in Eq. (2). The effective dimension-five Lagrangian for neutrino masses is obtained after integrating out the N_α fields:

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \frac{H}{\Lambda} \ell_i [Y_{1i} (\psi_1 \phi_1^* + \psi_2 \phi_2^* + \psi_3 \phi_3^* + \psi_4 \phi_4^*) + Y_{2i} (\psi_1 \phi_4 - \psi_2 \phi_3 + \psi_3 \phi_2 - \psi_4 \phi_1)] \\ &+ \frac{Y_3}{\Lambda} (\psi_1 \phi_1^* + \psi_2 \phi_2^* + \psi_3 \phi_3^* + \psi_4 \phi_4^*)^2 + \frac{Y_4}{\Lambda} (\psi_1 \phi_4 - \psi_2 \phi_3 + \psi_3 \phi_2 - \psi_4 \phi_1)^2 \\ &+ \frac{Y_5}{\Lambda} (\psi_1 \phi_4 - \psi_2 \phi_3 + \psi_3 \phi_2 - \psi_4 \phi_1) (\psi_1 \phi_1^* + \psi_2 \phi_2^* + \psi_3 \phi_3^* + \psi_4 \phi_4^*) + \frac{Y_{ij}}{\Lambda} H^2 \ell_i \ell_j + \text{h.c.}, \end{aligned} \quad (4)$$

where Y_{ij} , Y_{1i} , Y_{2i} , Y_3 , Y_4 , and Y_5 ($i, j = e, \mu, \tau$) are dimensionless couplings related to $a_{i\alpha}$ and $b_{i\alpha}$ and $\Lambda \sim M_{ij}$. Here, the couplings Y_{1i} and Y_{2i} , for example, arise respectively from the terms $\sim a_{i\alpha} b_\alpha$ and $\sim a_{i\alpha} c_\alpha$ in Eq. (3). The most general dimension-five neutrino mass operators which arise by integrating out arbitrary fermion representations (*i.e.*, by integrating out $SU(2)$ spin $j = 1, 2, 3$ fermions in addition to the $j = 0$ states N_α) are given in Appendix A. These mass terms however, will not alter our general results here.

Following Appendix B, where the most general scalar potential for Φ has been minimized, we can assume a VEV of the form $\langle \Phi \rangle = (v_1, 0, 0, v_4)$, with v_1 and v_4 as given in Eqs. (17) and $v_1, v_4 \sim 10^2 \text{ GeV}$. Since $\langle \Phi \rangle$ breaks $SU(2)$ completely, the component-fields of Ψ will finally appear as four sterile neutrinos $(\nu'_1, \nu'_2, \nu'_3, \nu'_4) \equiv (\psi_1, \psi_4, \psi_2, \psi_3)$ in the low-energy theory (note in the definition the permutation of indices).

Integrating out the right-handed neutrinos N_α , the sterile neutrino seesaw mechanism leads to five light neutrinos with finite masses in the (sub-) eV-range and two massless neutrinos. The massless states are ν'_3 and ν'_4 which decouple from $\nu_e, \nu_\mu, \nu_\tau, \nu'_1$, and ν'_2 (this is actually independent of the total number of right handed neutrinos N_α). The vacuum respects an unbroken Z_3 symmetry, which is a subgroup of I_3 , under which ν'_3 and ν'_4 have charges ± 1 while the other fermionic fields are all neutral. This Z_3 symmetry forbids the mixing of ν'_3 and ν'_4 with the other neutrinos. These states will acquire (sub-) eV masses once the effective Lagrangian $\mathcal{L}'_{\text{eff}}$ in Eq. (14) is taken into account. The resulting non-vanishing 5×5 effective neutrino mass matrix can be written in the basis $(\nu_e, \nu_\mu, \nu_\tau, \nu'_1, \nu'_2)$ as

$$M_{\text{eff}} = \begin{pmatrix} \mathcal{M}_\nu & \mathcal{M}'_\nu \\ \mathcal{M}'_\nu{}^T & \mathcal{M}''_\nu \end{pmatrix}, \quad (5)$$

that are homogeneous of degree $2j$, which is a complex representation space.

where \mathcal{M}_ν is an arbitrary 3×3 matrix with entries of the order $\sim 10^{-2}$ eV, while \mathcal{M}'_ν is given by the 3×2 matrix

$$\mathcal{M}'_\nu = \frac{\langle H \rangle}{\Lambda} \begin{pmatrix} Y_{1e}v_1^* + Y_{2e}v_4 & Y_{1e}v_4^* - Y_{2e}v_1 \\ Y_{1\mu}v_1^* + Y_{2\mu}v_4 & Y_{1\mu}v_4^* - Y_{2\mu}v_1 \\ Y_{1\tau}v_1^* + Y_{2\tau}v_4 & Y_{1\tau}v_4^* - Y_{2\tau}v_1 \end{pmatrix}, \quad (6)$$

and the 2×2 matrix \mathcal{M}''_ν reads

$$\mathcal{M}''_\nu = \frac{Y_3}{\Lambda} \begin{pmatrix} v_1^2 & v_1^*v_4^* \\ v_1^*v_4^* & v_4^2 \end{pmatrix} - \frac{Y_4}{\Lambda} \begin{pmatrix} -v_4^2 & v_1v_4 \\ v_1v_4 & -v_1^2 \end{pmatrix} + \frac{Y_5}{\Lambda} \begin{pmatrix} v_1^*v_4 & \frac{1}{2}(|v_4|^2 - |v_1|^2) \\ \frac{1}{2}(|v_4|^2 - |v_1|^2) & -v_1v_4^* \end{pmatrix}. \quad (7)$$

It is therefore seen that the effective interactions in Eq. (4) which generate the matrix \mathcal{M}'_ν introduce a non-zero mixing of ν'_1 and ν'_2 with the active neutrinos. Although the inclusion of the effective operators $\mathcal{L}'_{\text{eff}}$ in Eq. (14) lifts the zero neutrino masses to small values of the order $\Lambda^{-1}v_1v_2 \sim 1$ eV, the fields ν'_3 and ν'_4 will still remain decoupled from the rest of the neutrinos, owing to the unbroken Z_3 symmetry. In total, the model therefore gives in any case a (3+2) neutrino mass scheme for sterile neutrino oscillations.

3 Simple chiral $U(1)$ models

In Sec. 2, we have analyzed a simple gauge extension of G_{SM} to $G_{SM} \times SU(2)$. It is instructive to compare this model with a similar setup, where $SU(2)$ is replaced by a sterile $U(1)$ gauge symmetry to give the total gauge group $G_{SM} \times U(1)$. Let us therefore consider now N Weyl spinors Ψ_{n_i} ($i = 1, \dots, N$), where Ψ_{n_i} carries the charge n_i under the $U(1)$ gauge group. In this model, the anomaly cancellation conditions read $\sum_{i=1}^N n_i = 0$ (mixed gauge-gravitational anomaly) and $\sum_{i=1}^N n_i^3 = 0$ (cubic gauge anomaly). It is easy to see, that for $N \leq 4$ these conditions can only be fulfilled if the theory is vector-like, *i.e.*, the $U(1)$ model must contain at least five fermions to be chiral. Motivated by charge quantization, we shall require all charges n_i to be rational numbers, in which case they can be taken to be integers. Before discussing the case of $N = 5$ fermions, let us first consider simple chiral $U(1)$ models with $N = 6$. For this case, we find the following anomaly-free charge assignments:

$$\text{Model (a)} : 2 \times \{\mathbf{5}\} + 1 \times \{-\mathbf{3}\} + 1 \times \{-\mathbf{2}\} + 1 \times \{\mathbf{1}\} + 1 \times \{-\mathbf{6}\}, \quad (8a)$$

$$\text{Model (b)} : 2 \times \{\mathbf{4}\} + 3 \times \{-\mathbf{1}\} + 1 \times \{-\mathbf{5}\}. \quad (8b)$$

Here, Model (a), *e.g.*, has two Weyl fermions with $U(1)$ charge 5 and one state each with charge $-3, -2, 1$, and -6 . For Model (a), we minimally extend the Higgs sector by adding a single scalar singlet field Φ with $U(1)$ charge -5 . From the charge assignment in Eq. (8a) we then obtain the effective interaction Lagrangian for the neutrinos

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda} \ell_i H \Psi_5^\alpha \Phi + \frac{1}{\Lambda} \ell_j H H + \frac{1}{\Lambda} \Psi_5^\alpha \Psi_5^\beta \Phi \Phi + \Psi_{-3} \Psi_{-2} \Phi^* + \Psi_1 \Psi_{-6} \Phi^* + \text{h.c.}, \quad (9)$$

where $i = e, \mu, \tau$ and $\alpha, \beta = 1, 2$ and the Yukawa couplings have not been explicitly displayed. Similar to the $SU(2)$ model in Sec. 2, we suppose that Φ acquires its VEV at the TeV scale.

Hence, $\Psi_1, \Psi_{-2}, \Psi_{-3}$, and Ψ_{-6} will decouple below the TeV scale and we are left at low energies with a (3+2) model which is similar to the $SU(2)$ model.

For Model (b), a minimal extension of the Higgs sector by a scalar Φ with charge -4 leads to the effective neutrino mass Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda} \ell_i H \Psi_4^\alpha \Phi + \frac{1}{\Lambda} \ell_i \ell_j H H + \frac{1}{\Lambda} \Psi_4^\alpha \Psi_4^\beta \Phi \Phi + \text{h.c.}, \quad (10)$$

where $\alpha, \beta = 1, 2$. This gives essentially a (3+2) model with four additional extremely light neutrinos (the fields with charges -1 and -5) which decouple from the active neutrinos. When Φ , instead, carries the charge $+1$ we have the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda} \ell_i H \Psi_{-1}^\alpha \Phi + \frac{1}{\Lambda} \ell_i \ell_j H H + \frac{1}{\Lambda} \Psi_{-1}^\alpha \Psi_{-1}^\beta \Phi \Phi + \Psi_4^\gamma \Psi_{-5} \Phi + \text{h.c.}, \quad (11)$$

where $\alpha, \beta = 1, 2, 3$ and $\gamma = 1, 2$. These operators give rise to a (3+3) scheme with one additional massless neutrino (a linear combination of Ψ_4^γ) and two heavy neutrinos (Ψ_{-5} and one linear combination of Ψ_{-1}^γ) which all decouple.

Let us now consider the case of $N = 5$ fermions. In Diophantine analysis² it has been shown that every integer $n \neq \pm 4 \pmod{9}$ can be expressed as a sum of the cubes of four integers [23]. The integers $n = \pm 8 \pmod{18}$, for example, can be written as

$$(k - 5)^3 + (-k + 14)^3 + (3k - 30)^3 + (-3k + 29)^3 = 18k + 8 \quad (k \in \mathbb{Z}). \quad (12)$$

Choosing in Eq. (12) the value $k = 28$, we arrive at the integer solution $(n_1, n_2, n_3, n_4, n_5) \equiv (23, -14, 54, -55, -8)$ of the cubic anomaly cancellation condition. Note that none of the charges is vector-like. Simultaneously, this solution also gives a zero mixed gauge-gravitational anomaly. As a result, the simplest anomaly-free chiral $U(1)$ theory with only rational charges is given by

$$\text{Model (c)} : 1 \times \{\mathbf{23}\} + 1 \times \{-\mathbf{14}\} + 1 \times \{\mathbf{54}\} + 1 \times \{-\mathbf{55}\} + 1 \times \{-\mathbf{8}\}. \quad (13)$$

In comparison with the $N = 6$ models (a) and (b) in Eqs. (8), however, the charges in Eq. (13) involve rather large numbers, which makes this model less attractive.

4 Discussion

There are several experimental signatures of our models for naturally light sterile neutrinos. Generally speaking, the most striking consequences will be in the neutrino sector with very little effect elsewhere.

First, a confirmation of the LSND neutrino anomaly by MiniBooNE will clearly give credence to this class of models. Second, since a (3+2) neutrino mass scheme requires $U_{e5} \simeq 0.07$ [15], the model can be tested in the future by $\bar{\nu}_e$ (or ν_e) disappearance experiments. Moreover, with a fifth neutrino mass eigenvalue m_5 in the range $m_5 \sim 4 - 6 \text{ eV}$, the effective

²This is a subject which is mainly concerned with the discussion of the rational or integer solutions of a polynomial equation $f(n_1, n_2, \dots, n_N) = 0$ with integer coefficients.

Majorana mass in neutrinoless double β -decay $|\langle m \rangle|$ receives a contribution of the order ~ 0.02 eV, which has a good chance to be tested in next generation neutrinoless double β -decay experiments like GENIUS, EXO, MAJORANA, and MOON, which will have a sensitivity for $|\langle m \rangle| \sim 0.01$ eV.

Due to the non-zero mixing of H and Φ , the SM Higgs will have invisible decay modes such as $H \rightarrow \Phi\Phi$ and $H \rightarrow W'W'$, if these decays are kinematically allowed. This can be tested at LHC or a future linear collider.

Clearly, the requirement $N_\nu < 4$ on the total number of neutrino species N_ν from ${}^4\text{He}$ abundance in standard big bang nucleosynthesis (BBN) [24] is violated, since in all our schemes ν' will thermalize. However, there are suggestions that a primordial lepton asymmetry will weaken this bound [25]. Similarly, the neutrino mass limit $\sum m_\nu < 0.7 - 1.0$ eV (@95% C.L.) from recent cosmological data [26] may also be avoided for a suitable primordial ν_e chemical potential [27]. Our viewpoint here is, that if the (3+2) neutrino oscillation scheme is indeed confirmed by MiniBooNE, one will have to revise the standard BBN paradigm.

Finally, it has been suggested that a sterile neutrino in the 1–20 keV range with very small mixing ($\sin^2\theta \sim 10^{-11}$ – 10^{-7} for ν' - ν_e mixing) with the active neutrinos can serve as a possible dark matter candidate and may be responsible for the observed pulsar velocities exceeding ~ 500 km/sec [28]. Our models are readily adaptable to such a scenario.

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A Effective mass operators

Apart from the mass terms in Eq. (4), there exists in general a second type of effective dimension-five neutrino mass operators, which arise by integrating out arbitrary fermionic $SU(2)$ representations. The most general Lagrangian of these interactions reads

$$\begin{aligned} \mathcal{L}'_{\text{eff}} = & \frac{Y_6}{\Lambda} \left[2 \left(\psi_1 \psi_4 - \frac{1}{3} \psi_2 \psi_3 \right) \left(\phi_1 \phi_4 - \frac{1}{3} \phi_2 \phi_3 \right) - \frac{4}{3} \left(\frac{1}{\sqrt{3}} \psi_2^2 - \psi_1 \psi_3 \right) \left(\frac{1}{\sqrt{3}} \phi_3^2 - \phi_2 \phi_4 \right) \right. \\ & - \frac{4}{3} \left(\frac{1}{\sqrt{3}} \psi_3^2 - \psi_2 \psi_4 \right) \left(\frac{1}{\sqrt{3}} \phi_2^2 - \phi_1 \phi_3 \right) \Big] + \frac{Y_7}{\Lambda} \left[\frac{4}{3} \left(\frac{1}{\sqrt{3}} \psi_2^2 - \psi_1 \psi_3 \right) \left(\frac{1}{\sqrt{3}} \phi_2^{*2} - \phi_1^* \phi_3^* \right) \right. \\ & + 2 \left(\psi_1 \psi_4 - \frac{1}{3} \psi_2 \psi_3 \right) \left(\phi_1^* \phi_4^* - \frac{1}{3} \phi_2^* \phi_3^* \right) + \frac{4}{3} \left(\frac{1}{\sqrt{3}} \psi_3^2 - \psi_2 \psi_4 \right) \left(\frac{1}{\sqrt{3}} \phi_3^{*2} - \phi_2^* \phi_4^* \right) \Big] \\ & + \frac{Y_8}{\Lambda} \left[\frac{2}{3} \left(\frac{1}{\sqrt{3}} \psi_2^2 - \psi_1 \psi_3 \right) \left(\phi_1^* \phi_2 + \frac{2}{\sqrt{3}} \phi_2^* \phi_3 + \phi_3^* \phi_4 \right) + (|\phi_1|^2 + \frac{1}{3} |\phi_2|^2 - \frac{1}{3} |\phi_3|^2 - |\phi_4|^2) \right. \\ & \times \left. \left(\psi_1 \psi_4 - \frac{1}{3} \psi_2 \psi_3 \right) - \frac{2}{3} \left(\frac{1}{\sqrt{3}} \psi_3^2 - \psi_3 \psi_4 \right) \left(\phi_1 \phi_2^* + \frac{2}{\sqrt{3}} \phi_2 \phi_3^* + \phi_3 \phi_4^* \right) \right] + \text{h.c.}, \end{aligned} \quad (14)$$

where Y_6, Y_7 , and Y_8 denote Yukawa couplings of order unity. The most general effective neutrino mass operators are thus given by the sum $\mathcal{L}_{\text{eff}} + \mathcal{L}'_{\text{eff}}$. The gauge singlets in Eq. (14) can be determined from a Clebsh-Gordan table or by representing Ψ as a totally symmetric

tensor ψ_{ijk} , where $i, j, k = 1, 2$ and the (normalized) components are defined as $\psi_{111} = \psi_1$, $\psi_{112} = \psi_{121} = \psi_{211} = \frac{1}{\sqrt{3}}\psi_2$, $\psi_{122} = \psi_{212} = \psi_{221} = \frac{1}{\sqrt{3}}\psi_3$, and $\psi_{222} = \psi_4$ (correspondingly for Φ). In this notation, the coupling $\sim Y_7$, *e.g.*, can be obtained from the term $\psi_{abc}\psi_{ijk}\phi^{abi}\phi^{cjk}$ (summation of indices understood).

B Properties of the scalar potential

The most general renormalizable scalar potential of a $SU(2)$ spin 3/2 Higgs representation $\Phi = (\phi_1, \phi_2, \phi_3, \phi_4)^T$ is given by

$$\begin{aligned}
V = & -\mu^2 (|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2) + \lambda_1 (|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2)^2 \\
& + \lambda_2 \left(\left| \sqrt{\frac{2}{3}}\phi_1\phi_3 - \frac{\sqrt{2}}{3}\phi_2^2 \right|^2 + \left| \phi_1\phi_4 - \frac{1}{3}\phi_2\phi_3 \right|^2 + \left| \sqrt{\frac{2}{3}}\phi_2\phi_4 - \frac{\sqrt{2}}{3}\phi_3^2 \right|^2 \right) \\
& + \lambda_3 \left[2 \left(\sqrt{\frac{2}{3}}\phi_1\phi_3 - \frac{\sqrt{2}}{3}\phi_2^2 \right) \left(\sqrt{\frac{2}{3}}\phi_2\phi_4 - \frac{\sqrt{2}}{3}\phi_3^2 \right) - (\phi_1\phi_4 - \frac{1}{3}\phi_2\phi_3)^2 \right] \\
& + \lambda_4 \left[\phi_1^* \left(\phi_1^2\phi_4 + \frac{2}{3\sqrt{3}}\phi_2^3 - \phi_1\phi_2\phi_3 \right) + \phi_2^* \left(\phi_1\phi_2\phi_4 - \frac{2}{\sqrt{3}}\phi_1\phi_3^2 + \frac{1}{3}\phi_2^2\phi_3 \right) \right. \\
& \left. + \phi_3^* \left(-\phi_1\phi_3\phi_4 - \frac{1}{3}\phi_2\phi_3^2 + \frac{2}{\sqrt{3}}\phi_2^2\phi_4 \right) + \phi_4^* \left(\phi_2\phi_3\phi_4 - \frac{2}{3\sqrt{3}}\phi_3^3 - \phi_1\phi_4^2 \right) \right] + \text{h.c.}, \quad (15)
\end{aligned}$$

where the coefficients $\mu, \lambda_1, \lambda_2$, and λ_3 are real-valued³ and $\lambda_4 = |\lambda_4| \cdot \exp(i\beta)$ with some arbitrary phase β . Notice that the potential V possesses the following $U(1)$ symmetry which is part of the $SU(2)$ symmetry and allows to set one phase of the fields ϕ_i always to zero:

$$U(1) : \quad \phi_1 \rightarrow e^{+i\varphi}\phi_1, \quad \phi_2 \rightarrow e^{+i\varphi/3}\phi_2, \quad \phi_3 \rightarrow e^{-i\varphi/3}\phi_3, \quad \phi_4 \rightarrow e^{-i\varphi}\phi_4. \quad (16)$$

The potential V has a local extremum of the form $\langle \Phi \rangle = (v_1, 0, 0, v_4)$, where the complex entries v_1 and v_4 have a relative phase α , *i.e.*, it is $v_1v_2 = |v_1v_2| \cdot \exp(i\alpha)$. For simplicity, we may consider the limit $|\lambda_4| \ll 1$, in which case these quantities can be expressed to leading order as

$$|v_1|^2 \simeq \frac{\mu^2}{4\lambda_1 + \lambda_2 - 2|\lambda_3|} \left(1 \pm \frac{2|\lambda_4|}{\lambda_2 - 2|\lambda_3|} \cos(\beta) \right), \quad (17a)$$

$$\left| \frac{v_4}{v_1} \right| \simeq 1 \pm \frac{2|\lambda_4|}{\lambda_2 - 2|\lambda_3|} \cos(\beta), \quad (17b)$$

$$\alpha \simeq \frac{|\lambda_4|^2 \sin(2\beta)}{\lambda_3(\lambda_2 - 2\lambda_3)}. \quad (17c)$$

Notice in Eq. (15) that each interaction involves either zero, two, or four of the fields ϕ_2 and/or ϕ_3 . In the minimum $(v_1, 0, 0, v_4)$, the mixing of ϕ_2 and ϕ_3 with ϕ_1 and ϕ_4 is hence zero. As a consequence, the mass matrix of ϕ_2 and ϕ_3 has one pair of zero eigenvalues which correspond to two (would-be) Nambu-Goldstone bosons and two degenerate non-zero mass-squared eigenvalues of the form

$$m_{H^\pm}^2 = \frac{2}{3} (|v_1|^2 + |v_4|^2) \left(\lambda_2 + 6 \frac{|\lambda_4 v_1 v_4| \cos(\alpha)}{|v_4|^2 - |v_1|^2} \right). \quad (18)$$

³The phase of λ_3 can always be removed by an appropriate phase-redefinition $\Phi \rightarrow e^{i\varphi}\Phi$.

To calculate the remaining scalar masses, we consider the fluctuations $\phi_1 = v_1 + \tilde{\phi}_1$, and $\phi_4 = v_4 + \tilde{\phi}_4$ about the minimum $(v_1, 0, 0, v_2)$. The corresponding mass eigenstates G, A, H_1 , and H_2 can be expressed as

$$G = \frac{\sqrt{2} \operatorname{Im}(v_1^* \tilde{\phi}_1 - v_4^* \tilde{\phi}_4)}{\sqrt{|v_1|^2 + |v_2|^2}}, \quad A = \frac{\sqrt{2} \operatorname{Im}(v_4 \tilde{\phi}_1 + v_1 \tilde{\phi}_4)}{\sqrt{|v_1|^2 + |v_2|^2}}, \quad (19a)$$

$$H_1 = \frac{\sqrt{2} \operatorname{Re}(v_1^* \tilde{\phi}_1 - v_4^* \tilde{\phi}_4)}{\sqrt{|v_1|^2 + |v_2|^2}}, \quad H_2 = \frac{\sqrt{2} \operatorname{Re}(v_4 \tilde{\phi}_1 + v_1 \tilde{\phi}_4)}{\sqrt{|v_1|^2 + |v_2|^2}}. \quad (19b)$$

The scalar G is a massless (would-be) Nambu-Goldstone boson which has zero mixing with the other fields. In the limit $|\lambda_4| \ll 1$, the 3×3 mixing matrix of the fields A, H_1 , and H_2 has the mass-squared eigenvalues

$$m_{1,2}^2 \simeq (2\lambda_1 + \lambda_2)(|v_1|^2 + |v_2|^2) \pm \sqrt{(2\lambda_1 + \lambda_2)^2(|v_1|^2 + |v_2|^2)^2 - 8\lambda_1\lambda_2(|v_1|^2 - |v_2|^2)}, \quad (20a)$$

$$m_3^2 \simeq + \frac{|\lambda_4|}{|v_1 v_4|} (|v_1|^2 + |v_2|^2)(|v_1|^2 - |v_2|^2) \cos(\beta). \quad (20b)$$

In total we see, that for a range of parameters the extremum described in Eqs. (17) will be a local minimum. In this minimum, the $SU(2)$ gauge symmetry is completely broken, thereby leaving three (would-be) Nambu Goldstone bosons, which must be eaten by the gauge bosons via the Higgs mechanism. The kinetic term of Φ is obtained from the covariant derivative

$$D_\mu \phi_{ijk} = \partial_\mu \phi_{ijk} - i \frac{g'_2}{2} [(W'_\mu)_i^\alpha \phi_{\alpha jk} + (W'_\mu)_j^\alpha \phi_{i\alpha k} + (W'_\mu)_k^\alpha \phi_{ij\alpha}],$$

where g'_2 is the gauge coupling and $(W'_\mu)_i^l$ ($i, l = 1, 2$) are the $SU(2)$ gauge bosons. In the minimum $\langle \Phi \rangle = (|v_1|, 0, 0, |v_4| \cdot e^{i\alpha})$, the gauge boson masses are

$$m_{W'_3}^2 = \frac{9}{2} g_2'^2 (|v_1|^2 + |v_4|^2) = 3m_{W'_\pm}^2. \quad (21)$$

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